

within the noisy blurred scene data (D1). The reconstructed object, $G(x)$, which has been super-resolved, is then output as shown in module 70.

The background subtraction method for super-resolving the object of interest in D1 is detailed in FIG. 11. In FIG. 11 the noisy blurred scene data D1 containing the object to be reconstructed is used as input, with a low-pass filter (block 600) applied to remove high spatial frequency noise from the D1 data as shown by equation 11(a). In equation 11(a) the transfer function $h_b(x)$ represents the Fourier transform of the binoff array. The binoff array specifies the non-zero spatial frequency of the of located in the fourier plane. FIG. 12 shows the fourier transform of an image depicting binoff to ensure no higher frequencies in the spectrum exist.

As shown in FIG. 12, the values of binoff are 1 up to the optical system cutoff value f_c and 0 beyond that cutoff. The low pass filtered data $D_f(x)$ in FIG. 11 is then multiplied pixel by pixel with the bin map array (binmap (x)) to separate out a first estimate of the reconstructed object from the filtered D1 data, as shown in equation 11(b) in module 610. The binmap array specifies the region in the scene containing the object to be super-resolved. The binmap has array elements equal to 1 where the object of interest is located and array elements equal to 0 everywhere else. FIG. 13 shows a pictorial representation of the binmap array, where the binmap window (40) holds a region containing an object of interest (20) consisting of pixels equal to one in the region containing the object and pixels equal to zero everywhere else (30).

The next step in FIG. 11 is to replace the reconstructed background scene pixels, $I_r(x)$, by the estimated reconstructed object pixels, $D_o(x)$, at object positions specified by the binmap (40), as shown in module 620. Equation 11(c) provides the mathematical formula for this replacement, creating a reconstructed object array $S(x)$. $S(x)$ is then convolved with the optical system PSF (h_o) to blur the combination of the reconstructed background and the estimated reconstructed object as shown in equation 11(d) of module 630. A new array, $N(X)$, is then created in block 640 by dividing, on a pixel by pixel basis, the filtered D1 scene array (D_f) by the blurred combination of the reconstructed background and the estimated reconstructed object (I_r) as shown in equation 11(e) of module 640. The new array, $N(x)$, is then correlated with the optical system PSF (h_o) and multiplied, for each pixel specified by binmap, by the current estimate of the reconstructed object. Equation 11(f) of module 650 is then used to determine $K(x)$, the new estimate of the reconstructed object.

After $K(x)$ has been calculated a check is made, as shown in module 660, to determine whether the specified number of iterations have been completed. If more iterations are needed the current estimate of the reconstructed object is replaced by the new estimate as shown by equation 11(g) of module 670. Steps 620–660 are repeated until the specified number of iterations have been accomplished. When this happens the latest estimate of the reconstructed objects is taken to be the reconstructed object; that is $G(x)$ is set equal to $K(x)$, as shown in module 680 (equation 11H).

In FIG. 9B the reconstructed object $G(x)$ is the output of module 70. This $G(x)$ is in fact the desired super-resolved object. Note that the above description of the super-resolution method as shown in FIG. 11 is set up to handle non-thinned apertures. For thinned aperture systems, step 630 of FIG. 11 (in which the new scene $I_r(x)$ is blurred again using the optical system's PSF) may be excluded.

Three examples of applying the background reconstruction approach using the above-described non-linear tech-

nique to obtain super-resolution are illustrated in FIGS. 14–16. FIGS. 14A–D represent simulated bar target charts where FIG. 14A represents the truth scene illustrated by a series of alternating dark and light bands within a background. FIG. 14B represents the blurred image of the truth scene (through a small aperture), and FIGS. 14C and 14D represent the reconstructed images using the non-linear background reconstruction method previously described. FIGS. 15A–D are associated respectively with FIGS. 14A–D and represent a one-dimensional Fourier transform cut through each of the “scenes”, thus clearly illustrating the spatial frequencies that have been restored to the reconstructed image.

FIGS. 16A–C represent the application of the non-linear method to a thinned aperture system. In this case, the thinned aperture configuration is an annulus. It should be noted, however, that the method may be utilized with any thinned aperture design. FIG. 16A represents a computer generated ground scene (i.e. the truth scene). The blurred image of that scene is then depicted in FIG. 16B, while the final reconstructed, super-resolved image is shown in FIG. 16C.

FIGS. 17A–C represent images of figures taken from a CCD camera. FIG. 17A represents the truth scene (a picture of a toy spaceman), while FIG. 17B shows the blurred image of the scene (observed through a small aperture). FIG. 17C represents the reconstructed image, and FIG. 17D shows the magnitude of the difference of the two-dimensional Fourier transform between the truth scene in FIG. 17A and the blurred image of FIG. 17B. FIG. 17E shows the difference between the truth scene and the first stage of reconstruction (i.e. the deconvolved figure), while FIG. 17F shows the magnitude of the difference of the two-dimensional Fourier transform and the truth scene for the reconstructed, super resolved image. Note that black indicates a 0 difference, which is the desired result, while white indicates a maximum difference. As one can see from a comparison of FIGS. 17D, E and F, the radius of FIG. 17D corresponds to the cutoff of the optical system or camera, and the deconvolved image frequencies in FIG. 17E have been enhanced inside the cutoff but remain zero outside the cutoff. The super-resolved figure in FIG. 17F has further improved the image by restoring frequencies outside the cutoff as can be shown by the increased blackness of the figure with respect to either FIGS. 17D or 17E. This is a clear demonstration that super resolution has occurred. FIGS. 18A–B represent a graphical illustration of the truth, blurred, and reconstructed super-resolved images for SNR values of 50 and 100 respectively. FIGS. 18A and B show that the non-linearly reconstructed images closely parallel the truth images.

In an alternative embodiment, the reconstruction approach using a linear transformed method is now described. When reconstructing either the background scene or the localized object, the imaging system is mathematically characterized by a linear operator represented by a matrix. To restore either a background scene or the localized object, an inverse imaging matrix corresponding to the inverse operator must be constructed. However, due to the existence of system noise, applying the inverse imaging matrix to the image is intrinsically unstable and results in a poorly reconstructed image. However by applying a constrained least squares procedure such as Tikhonov regularization, a regularized pseudo-inverse (RPI) matrix may be generated. Zero-order Tikhonov regularization is preferably used, although higher order Tikhonov regularization may sometimes give better results. The details of this are not described here, as Tikhonov regularization is well-known in the art.